

NOTE ON THE MINIMUM NUMBER OF CRITICAL POINTS  
OF REAL OR CIRCULAR FUNCTIONS

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**Abstract.** The aim of this note is to estimate the minimum number of critical points of circular functions, which are defined on a product of two manifolds, in terms of the minimum number of critical points of circular functions, defined on the two factors. Some special attention is paid to the class of circular Morse functions. Such estimations were previously done for real functions in [2].

**MSC 2010.** 57R70, 58E05.

**Key words.** Circular function,  $\varphi$ -category, circular  $\varphi$ -category, Morse function.

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Received May 5, 2017  
Accepted May 15, 2017

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