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A GENERALIZATION OF STEFFENSEN'S INTEGRAL INEQUALITY FOR THE SUGENO INTEGRAL

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Abstract. In this paper we prove the Bellman generalization of Steffensen's integral inequality for the Sugeno integral

$$\left(\int_0^1 f g \mathrm{d}\mu\right)^p \leq \int_0^\lambda f^p \mathrm{d}\mu,$$

where f is a nonincreasing and left continuous function defined on $[0,1]$ with $f(0) = 1$, $f(1) = 0$, g is a nonincreasing function defined on $[0,1]$ with $0 \leq g(t) \leq 1$, for all $t \in [0,1]$, $\lambda = \left(\int_0^1 g \mathrm{d}\mu\right)^p$, and $p \geq 1$.

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Key words. Sugeno integral, fuzzy measure, Bellman's integral inequality.

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