SOME INTEGRAL INEQUALITIES OF HERMITE-HADAMARD TYPE FOR *GG*-CONVEX FUNCTIONS

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Abstract. Some related integral inequalities of Hermite-Hadamard type for *GG*-convex functions defined on positive intervals are given. Applications for the exponential integral mean are also provided.

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Key words. Convex function, integral inequality, GG-Convex function, Hermite-Hadamard type inequality.

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