

SOME CATEGORICAL ASPECTS
IN TOPOLOGICAL FORMAL CONCEPT ANALYSIS

BRIGITTE E. BRECKNER and CHRISTIAN SĂCĂREA

Abstract. Formal Concept Analysis (FCA) is a prominent field of Applied Mathematics which is grounded on the mathematization of the notion of concept and concept hierarchy, having a wide range of applications in data analysis and knowledge discovery in databases. Topological FCA is investigating issues related to the interplay between Topology and FCA. This paper is devoted to the study of some categorical equivalences in topological FCA. We prove that the category of pseudometric contexts is dually equivalent to a certain category of complete lattices enhanced with a pseudometric, extending by this the Basic Theorem on Concept Lattices to topological FCA.

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Babeş-Bolyai University
Faculty of Mathematics and Computer Science
1 M. Kogălniceanu St.
400084 Cluj-Napoca, Romania
E-mail: brigitte@math.ubbcluj.ro
E-mail: csacarea@math.ubbcluj.ro