

NECESSARY AND SUFFICIENT CONDITIONS FOR
OSCILLATION OF NONLINEAR NEUTRAL FIRST ORDER
DIFFERENTIAL EQUATIONS WITH SEVERAL DELAYS

SHYAM SUNDAR SANTRA

Abstract. In this work, necessary and sufficient conditions for oscillations of the solutions of a class of nonlinear first-order neutral differential equations with several delays of the form

$$(x(t) + r(t)x(t - \tau))' + \sum_{i=1}^m \phi_i(t)H(x(t - \sigma_i)) = 0$$

are established under various ranges of $r(t)$. Finally, two illustrating examples are presented to show the feasibility and the effectiveness of the main results.

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Key words. Oscillation, nonoscillation, non-linear, delay, neutral differential equations, Knaster-Tarski fixed point theorem, Banach's fixed point theorem.

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Sambalpur University
Department of Mathematics
Sambalpur 768019, India
E-mail: shyam01.math@gmail.com