

SOLVABILITY FOR A NONLINEAR FOURTH-ORDER
THREE-POINT BOUNDARY VALUE PROBLEM

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Abstract. In this paper, we study the existence of a nontrivial solution for the fourth-order three-point boundary value problem having the following form

$$\begin{aligned} u^{(4)}(t) + f(t, u(t)) &= 0, \quad 0 < t < 1, \\ u(0) = 0, \quad u''(0) = u'''(0) &= 0, \quad u'(1) = \alpha u'(\eta), \end{aligned}$$

where $\eta \in (0, 1)$, $\alpha \in \mathbb{R}$, $\alpha \neq 1$, $f \in C([0, 1] \times \mathbb{R}, \mathbb{R})$. By using the Leray-Schauder nonlinear alternative, we prove the existence of at least one solution of the above problem. As an application, we also given some examples to illustrate the obtained results.

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Key words. Solvability, Green's function, Leray-Schauder nonlinear alternative, fixed point theorem, boundary value problem.

REFERENCES

- [1] AGARWAL, R.P., *Boundary Value Problems for Higher Order Differential Equations*, Institute of Mathematical Sciences, Madras, 1979.
- [2] DEIMLING, K., *Nonlinear Functional Analysis*, Springer, Berlin, 1985.
- [3] ELOE, P.W. and AHMAD, B., *Positive solutions of a nonlinear imageith order boundary value problem with nonlocal conditions*, Appl. Math. Lett., **18** (2005), 521–527.
- [4] GRAEF, J.R. and MOUSSAOUI, T., *A class of nth-order BVPs with nonlocal conditions*, Comput. Math. Appl, **58** (2009), 1662–1671.
- [5] JI, Y. and GUO, Y., *The existence of countably many positive solutions for some nonlinear nth order m-point boundary value problems*, J. Comput. Appl. Math., **232** (2009), 2, 187–200.
- [6] KRASNOSEL'SKII, M.A., *Topological Methods in the Theory of Nonlinear Integral Equations*, Pergamon Press, Oxford, 1964.
- [7] LI, S. and SUN, Y.P., *Nontrivial solution of a nonlinear second-order three-point boundary value problem*, Appl. Math. J. Chinese Univ. Ser. B, **22** (2007), 1, 37–47.
- [8] LIU, Y. and GE, W., *Positive solutions for $(n-1, 1)$ three-point boundary value problems with coefficient that changes sign*, J. Math. Anal. Appl., **282** (2003), 2, 816–825.
- [9] SUN, Y.P., *Nontrivial solution for a three-point boundary-value problem*, Electron. J. Differential Equations, **2004** (2004), 111, 1–10.
- [10] SUN, Y.P., *Nontrivial Symmetric Solution of a Nonlinear Second-Order Three-Point Boundary Value Problem*, Miskolc Math. Notes, **10** (2009), 1, 97–106.

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- [11] XIE, D., LIU, Y. and BAI, C., *Green's function and positive solutions of a singular n th-order three-point boundary value problem on time scales*, Electron. J. Qual. Theory Differ. Equ., **2009** (2009), 38, 1–14.
- [12] YANG, J. and WEI, Z., *Positive solutions of n th order m -point boundary value problem*, Appl. Math. Comput., **202** (2008), 715–720.

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