

WITT OVERGROUPS FOR UNIPOTENT ELEMENTS
IN EXCEPTIONAL ALGEBRAIC GROUPS
OF BAD CHARACTERISTIC

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Abstract. Let G be a simple exceptional algebraic group defined over an algebraically closed field of bad characteristic. The decompositions as a product of Witt groups of the connected component of the double centralizer $Z(C_G(u))^\circ$ for unipotent elements u is given up to isogeny. For type G_2 , F_4 and E_6 minimal dimensional connected overgroups for unipotent elements are constructed in G whenever $u \in C_G(u)^\circ$.

MSC 2010. 20G15, 17B45.

Key words. Exceptional algebraic group of bad characteristic, Witt group, isogeny.

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The author was supported by the Swiss National Science Foundation through grants 200021-122267 and 200020-135144 as well as by the grant CPDR131579/13 of Padova University.

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Received September 22, 2015

Accepted November 14, 2015

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