

## ON BESSEL-MAITLAND MATRIX FUNCTION

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**Abstract.** The main object of this paper is to consider the Bessel-Maitland matrix function in the following form:

$$\phi(A, B; z) = \sum_{k=0}^{\infty} \frac{z^k}{k!} \Gamma^{-1}(kA + B).$$

A different approach is adopted to study the radius of regularity, order and type of this function. Certain properties including integral representation and differential recurrence relations are also derived. Further, the composite Bessel-Maitland matrix function is introduced and its properties are discussed.

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**Key words.** Hypergeometric matrix function, Bessel-Maitland matrix function, integral representation.

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