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OSCILLATION CRITERIA FOR NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS OF FIRST ORDER WITH SEVERAL DELAYS

SHYAM SUNDAR SANTRA

Abstract. In this work, oscillatory behaviour of the solutions of a class of non-linear first-order neutral differential equations with several delays of the form

$$(E_1) \qquad (x(t) + p(t)x(t-\tau))' + \sum_{i=1}^{m} q_i(t)H(x(t-\sigma_i)) = f(t)$$

and

$$(E_2) \qquad (x(t) + p(t)x(t-\tau))' + \sum_{i=1}^{m} q_i(t)H(x(t-\sigma_i)) = 0$$

are studied under various ranges of $p(t)$. Sufficient conditions are obtained for existence of bounded positive solutions of (E_1).

MSC 2010. 34C10, 34C15, 34K40.

Key words. Oscillation, nonoscillation, non-linear, delay, neutral differential equations, existence of positive solution.

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Sambalpur University
Department of Mathematics
Sambalpur – 768019, India
E-mail: shyam01.math@gmail.com