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SOME TYPES OF DERIVATIONS ON HILBERT C*-MODULES AND THEIR OPERATOR ALGEBRAS

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Abstract. Let $\mathcal{A}$ be a C^*-algebra and $\mathcal{M}$ be a Hilbert $\mathcal{A}$-module. In this paper, we show that if $\mathcal{A}$ is commutative and there exist $x_0, y_0 \in \mathcal{M}$ such that $\langle x_0, y_0\rangle = 1_{\mathcal{A}}$, then every Jordan ternary derivation on $\mathcal{M}$ is a ternary derivation. Moreover, motivated by definition of Jordan $*$-derivations, we study innerness of the linear mapping J on $\mathrm{End}^*_{\mathcal{A}}(\mathcal{M})$, the C^*-algebra of adjointable operators in Hilbert C^*-modules satisfying $J(T^2) = J(T)T + T^*J(T)$ for all $T \in \mathrm{End}^*_{\mathcal{A}}(\mathcal{M})$. Also, motivated by definition of reverse $*$-derivations, some necessary conditions for mapping D on $\mathrm{End}^*_{\mathcal{A}}(\mathcal{M})$ satisfying $D(TS) = D(S)T^* + S^*D(T)$ for $T \in \mathrm{End}^*_{\mathcal{A}}(\mathcal{M})$ to be inner will be established. At the end of this paper, we characterize the linear mappings on $\mathrm{End}^*_{\mathcal{A}}(\mathcal{M})$ which behave like mapping D when acting on pairs of elements with compact product.

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Key words. Hilbert C^*-modules, ternary derivations, Jordan $*$-derivations, reverse $*$-derivations, innerness.

REFERENCES

[1] Abbaspour, Gh. and Skeide, A., *Generators of dynamical systems on Hilbert modules*, Commun. Stoch. Anal., **1** (2007), 193–207.

[2] Bakić, D. and Guljaš, B., *Hilbert C^*-modules over C^*-algebras of compact operators*, Acta Sci. Math. (Szeged), **68** (2002), 249–269.

[3] Battyányi, P., *On the range of a Jordan $*$-derivation*, Comment. Math. Univ. Carolin., **37** (1996), 659–665.

[4] Battyányi, P., *Jordan $*$-derivations with respect to the Jordan product*, Publ. Math. Debrecen, **48** (1996), 327–338.

[5] Brešar, M., *Jordan derivations on semiprime rings*, Proc. Amer. Math. Soc., **104** (1988), 1003–1006.

[6] Brešar, M. and Zalar, B., *On the structure of Jordan $*$-derivations*, Colloq. Math., **53** (1992), 163–171.

[7] Brešar, M. and Vukman, J., *On some additive mappings in rings with involution*, Aequationes Math., **38** (1989), 178–185.

[8] Brown, L.G., *Stable isomorphism of hereditary subalgebras of C^*-algebras*, Pacific J. Math., **71** (1977), 335–348.

[9] Herštein, I.N., *Jordan derivations of prime rings*, Proc. Amer. Math. Soc., **8** (1957), 1104–1110.

[10] Isidro, J.M., *Holomorphic automorphisms of the unit balls of Hilbert C^*-modules*, Glasgow Math. J., **45** (2003), 249–262.

[11] Kaplansky, I.,*Modules over operator algebras*, Amer. J. Math., **75** (1953), 839–858.

[12] Kafimoghadam, M., Miri, M. and Janfada, A.R., *A note on derivations on the algebra of operators in Hilbert C$*$-modules*, Mediterr. J. Math., **13** (2016), 1167–1175.

[13] Li, P.T., Han, D.G. and Tang, W.S., *Derivations on the algebra of operators in Hilbert C^*-modules*, Acta Math. Sinica, **28** (2012), 1615–1622.
[14] Liu, X. and Xu, T.Z., *Automatic continuity of derivations of Hilbert C^*-modules*, J. Baoji College Arts Sci. Nat. Sci., **2** (1995), 14–17.
[15] Manuilov, V.M. and Troitsky, E.V., *Hilbert C^*-modules*, Transl. Math. Monogr., Amer. Math. Soc., Providence, RI, **226** (2005).
[16] Moslehian, M.S. and Najati, A. *Jordan (ϕ, ψ)-derivations in JB^*-triples*, Commun. Korean Math. Soc., **26** (2011), 585–589.
[17] Molnár, L. and Šemrl, P., *Local Jordan $*$-derivation of standard operator algebras*, Proc. Amer. Soc., **125** (1997), 447–454.
[18] Molnár, L., *On the range of a normal Jordan $*$-derivation*, Comment. Math. Univ. Carolin., **35** (1994), 691–695.
[19] Šemrl, P., *Jordan $*$-derivations of standard operator algebras*, Proc. Amer. Math. Soc., **120** (1994), 515–518.
[20] Šemrl, P., *On Jordan $*$-derivations and an application*, Colloq. Math., **59** (1990), 241–251.
[21] Šemrl, P., *Quadratic functional and Jordan $*$-derivations*, Studia Math., **97** (1991), 157–165.
[22] Zalar, B., *Jordan $*$-derivation pairs and quadratic functional on modules over $*$-ring*, Aequationes Math., **54** (1997), 31–43.

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