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SEMIREGULAR MODULES RELATIVE TO A PRERADICAL

TAYYEBEH AMOUZEGAR

Abstract. Let τ_M be a preradical on the category $\sigma[M]$ for some module M. A module $N \in \sigma[M]$ is called τ_M-semiregular in $\sigma[M]$ if for all $n \in N$, there exists a decomposition $N = A \oplus B$ such that A is a projective submodule of nR and $nR \cap B \subseteq \tau_M(N)$. We prove that if $N \in \sigma[M]$ is a projective module, then N is τ_M-semiregular if and only if N is finitely τ_M-supplemented and that $\tau_M(N)$ is quasi finitely strongly lifting (for short QFSL) if and only if every finitely generated submodule of $N/\tau_M(N)$ is a direct summand and $\tau_M(N)$ is QFSL. Furthermore, it is shown that if $N \in \sigma[M]$ is a τ_M-semiregular module, then N is finitely refinable if and only if every submodule of $\tau_M(N)$ is QFSL in N if and only if every finitely generated submodule of $\tau_M(N)$ is DM in N.

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Key words. τ_M-semiregular modules; projective modules, τ_M-supplement submodules, finitely generated submodules.

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Quchan University
of Advanced Technology
Department of Mathematics
Quchan, Iran
E-mail: t.amoozegar@yahoo.com
E-mail: t.amouzgar@qiet.ac.ir