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AN ARCLENGTH PROBLEM FOR SOME SUBCLASSES OF m-FOLD SYMMETRIC UNIVALENT FUNCTIONS

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Abstract. For $0 < \beta \leq 1$, let $\mathcal{F}_m(\beta)$ (respectively $\mathcal{G}_m(\beta)$) denote the class of analytic functions f in the unit disk $\mathbb{D}$ with $f(0) = 0$, $f'(0) = 1$ and $f(\mathrm{e}^{\frac{2\pi\mathrm{i}}{m}} z) = \mathrm{e}^{\frac{2\pi\mathrm{i}}{m}} f(z)$ satisfying $\operatorname{Re} P_f(z) < \frac{\beta}{2} + 1$ (respectively $\operatorname{Re} P_f(z) > \frac{\beta}{2} - 1$) for $z \in \mathbb{D}$, where

$$P_f(z) = 1 + \frac{zf''(z)}{f'(z)}.$$

For $|\alpha| < \pi/2$, let $\mathcal{S}_\alpha$ denote the class of univalent functions $f(z)$ for which $zf'(z)$ is spirallike functions which has been introduced by M.S. Robertson [18]. The main aim of this paper is to investigate arclength problem

$$L_r(f) = \int_{|z|=r} |f'(z)|\,|\mathrm{d}z|, \quad 0 < r < 1$$

for functions f in $\mathcal{F}_m(\beta)$, $\mathcal{G}_m(\beta)$ and $\mathcal{S}_\alpha$. As a consequence, we shall obtain arclength for functions in some subclasses of the class of univalent functions. In each of these subclasses, we shall provide extremal functions to obtain the sharp upper bound for $L_r(f)$.

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