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D.D. STANCU OPERATORS: ON SOME OF THEIR LINEAR COMBINATIONS AND GENERALIZATIONS

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Abstract. More than forty years ago, in his paper [22] from 1968, academician professor D.D. Stancu introduced and studied a new sequence of linear and positive operators, $S_n^\alpha : C[0,1] \to C[0,1]$,

$$\left(S_n^\alpha f\right)(x) = \sum_{k=0}^{n} \omega_{(n,k)}^\alpha(x) f\left(\frac{k}{n}\right),$$

where

$$\omega_{(n,k)}^\alpha(x) = \binom{n}{k} \frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}},$$

$n \in \mathbb{N}$ and α is a real parameter depending only on n. We recall that $\omega_{(n,k)}^\alpha$ are known as "the fundamental polynomials of Stancu". This paper is concerned with linear combinations of the Stancu polynomials. The idea was inspired by O. Agratini's work from 1998 [1]. The present paper also describes other generalizations and the author summarizes various results, due to a number of authors, that are concerned with the Stancu operators.

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Key words. Approximation by positive linear operators, Stancu operators.

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This article is a tribute dedicated to academician professor Dimitrie D. Stancu (1927 – 2014), who was my teacher, my advisor, my inspiration. "*A good teacher is like a candle. It consumes itself to light the way for others...*" (Mustafa Kemal Atatürk).

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