

\mathcal{T} -NORMAL DECOMPOSITIONS OF MODULES

IULIU CRIVEI and LIANA CRIVEI

Abstract. Let τ be a hereditary torsion theory on the category of modules. A module A is called finitely τ -completely decomposable if it is a finite direct sum of τ -uniform τ -injective modules. For a submodule B of a module A , we show that the τ -injective envelope of A/B is finitely τ -completely decomposable if and only if B has a so-called τ -normal decomposition in A .

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Key words. Torsion theory, τ -injective module, τ -irreducible submodule, τ -uniform module, τ -completely decomposable module, τ -normal decomposition.

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Technical University
Department of Mathematics
Str. C. Daicoviciu 15
400020 Cluj-Napoca, Romania
E-mail: crivei@math.utcluj.ro

Babeş-Bolyai University
Faculty of Mathematics and Computer Science
Str. M. Kogălniceanu 1
400084 Cluj-Napoca, Romania