

DISSIPATIVE STURM-LIOUVILLE OPERATORS
ON BOUNDED TIME SCALES

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Abstract. In this paper we consider a second-order Sturm–Liouville operator of the form

$$l(y) := - \left[p(t) y^\Delta(t) \right]^\nabla + q(t) y(t)$$

on bounded time scales. In this study, we construct a space of boundary values of the minimal operator and describe all maximal dissipative, maximal accretive, selfadjoint and other extensions of the dissipative Sturm–Liouville operators in terms of boundary conditions. Using by methods of Pavlov [28–30], we proved a theorem on completeness of the system of eigenvectors and associated vectors of the dissipative Sturm–Liouville operators on bounded time scales.

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Key words. Time scales, Sturm–Liouville operator, Dilation, Dissipative operator, Completeness of the system of eigenvectors and associated vectors, Scattering matrix, Functional model, Characteristic function.

REFERENCES

- [1] AGARWAL, R.P., BOHNER, M. and LI W.T., *Nonoscillation and Oscillation Theory for Functional Differential Equations*, Pure Appl. Math., Dekker, Florida, 2004.
- [2] ALLAHVERDIEV B.P., *On dilation theory and spectral analysis of dissipative Schrödinger operators in Weyl’s limit-circle - case*, Izv. Akad. Nauk. SSSR Ser. Mat., **54** (1990), 242–257; English transl.: Math. USSR Izv., **36** (1991), 247–262.
- [3] ALLAHVERDIEV, B.P., *Spectral analysis of dissipative Dirac operators with general boundary conditions*, J. Math. Anal. Appl., **283** (2003), 287–303.
- [4] ALLAHVERDIEV, B.P., *Dilation and Functional Model of Dissipative Operator Generated by an Infinite Jacobi Matrix*, Math. and Comp. Modelling, **38** (2003), 989–1001.
- [5] ALLAHVERDIEV, B.P., *Dissipative second-order difference operators with general boundary conditions*, Journal of Difference Equations and Applications, **10**, (2004), 1–16.
- [6] ALLAHVERDIEV, B.P., *Spectral analysis of nonselfadjoint Schrödinger operators with a matrix potential*, J. Math. Anal. Appl., **303** (2005), 208–219.
- [7] ANDERSON, D. R. , GUSEINOV GUSEIN, SH. and HOFFACKER, J., *Higher-order self-adjoint boundary-value problems on time scales*, J. Comput. Appl. Math., **194** (2006), 309–342.
- [8] BEHRNDT, J., MALAMUD, M.M. and NEIDHARDT, H., *Scattering theory for open quantum systems with finite rank coupling*, Math. Phys. Anal. Geom., **10** (2007), 313–358.
- [9] BEHRNDT, J., MALAMUD, M.M. and NEIDHARDT, H., *Scattering matrices and Weyl functions*, Proc. London Math. Soc., **97** (2008), 568–598.
- [10] BEHRNDT, J., MALAMUD, M.M. and NEIDHARDT, H., *Trace formulae for dissipative and coupled scattering systems*, Operator Theory Advances Applications, **188** (2008), 49–87.

- [11] BEHRNDT, J., MALAMUDN M.M. and NEIDHARDT, H., *Finite rank perturbations, scattering matrices and inverse problems*, Operator Theory Advances Applications, **198** (2009), 61–85.
- [12] BARO, M. and NEIDHARDT, H., *Dissipative Schrödinger-type operators as a model for generation and recombination*, J. Math. Phys., **44** (2003), 2373–2401.
- [13] BARO, M., KAISER, H.-CHR., NEIDHARDT, H. and REHBERG, J., *Dissipative Schrödinger–Poisson systems*, J. Math. Phys., **45** (2004), 21–43.
- [14] BARO, M., KAISER, H.-CHR., NEIDHARDT, H. and REHBERG, J., *A Quantum Transmitting Schrödinger–Poisson System*, Rev. Math. Phys., **16** (2004), 281–330.
- [15] BOHNER, M. and PETERSON, A., *Dynamic Equations on Time Scales*, Birkhäuser, Boston, 2001.
- [16] BOHNER, M. and PETERSON, A. (Eds.), *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, 2003.
- [17] GORBACHUK, M.L. and GORBACHUK, V.I., *Boundary Value Problems for Operator Differential Equations*, Naukova Dumka, Kiev, 1984; English transl. 1991, Birkhauser Verlag.
- [18] GUSEINOV GUSEIN, SH., *Self-adjoint boundary value problems on time scales and symmetric Green's functions*, Turkish J. Math., **29** (2005), 365–380.
- [19] HILGER, S., *Analysis on measure chains—a unified approach to continuous and discrete calculus*, Results Math., **18** (1990), 1818–1856.
- [20] HUSEYNOV, A., *Limit point and limit circle cases for dynamic equations on time scales*, Hacet. J. Math. Stat., **39** (2010), 379–392.
- [21] KOCHUBEI, A. N., *Extensions of symmetric operators and symmetric binary relations*, Mat. Zametki, **17** (1975), 41–48; transl. in Math. Notes, **17** (1975), 25–28.
- [22] KUZHEL, A., *Characteristic Functions and Models of Nonselfadjoint Operators*, Kluwer Academic, Dordrecht, 1996.
- [23] LAKSHMIKANTHAM, V., SIVASUNDARAM, S. and KAYMAKCALAN, B., *Dynamic Systems on Measure Chains*, Kluwer Academic Publishers, Dordrecht, 1996.
- [24] LAX, P.D. and PHILLIPS, R.S., *Scattering Theory*, Academic Press, New York, 1967.
- [25] NAGY, B. SZ. and FOIAŞ, C., *Analyse Harmonique des Operateurs de L'espace de Hilbert*, Masson, Akad. Kiado, Paris, Budapest, 1967; English transl., North-Holland, Amsterdam, and Akad. Kiado, Budapest, 1970.
- [26] NAIMARK, M. A., *Linear Differential Operators*, 2nd edn., 1968, Nauka, Moscow; English transl. of 1st. edn., 1969, New York.
- [27] ATICI MERDIVENCI, F. and GUSEINOV GUSEIN, SH., *On Green's functions and positive solutions for boundary value problems on time scales*, J. Comput. Appl. Math., **141** (2002), 75–99.
- [28] PAVLOV, B.S., *Selfadjoint Dilation of a Dissipative Schrödinger Operator and Eigenfunction Expansion*, Funct. Anal. Appl., **98** (1975), 172–173.
- [29] PAVLOV, B.S., *Selfadjoint Dilation of a Dissipative Schrödinger Operator and its Resolution in terms of Eigenfunctions*, Math. USSR Sbornik, **31** (1977), 457–478.
- [30] PAVLOV, B.S., *Dilation theory and spectral analysis of nonselfadjoint differential operators*, Transl. Am. Math. Soc., **115** (1981), 103–142.
- [31] RYNNE, B.P., *L^2 spaces and boundary value problems on time-scales*, J. Math. Anal. Appl., **328** (2007), 1217–1236.

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