

SOME REMARKS ON UNIVERSAL COVERS AND GROUPS

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Abstract. We give a quick review of problems concerning the topological behavior of contractible covering spaces, from the point of view of the topology at infinity. In particular, we briefly describe the evolution of the notion of the simple connectivity at infinity, from low-dimensional topology to the asymptotic topology of discrete groups, also highlighting the work done by V. Poénaru on these topics.

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