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EXTREME POINTS AND SUPPORT POINTS FOR MAPPINGS WITH g-PARAMETRIC REPRESENTATION IN $\mathbb{C}^n$

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Abstract. We obtain various results related both to extreme points and to support points for the compact family $\overline{S_g^0(B^n)}$, where $S_g^0(B^n)$ is the family of normalized biholomorphic mappings which have g-parametric representation on the unit ball in $\mathbb{C}^n$, and g is a univalent function on the unit disc U with $g(0) = 1$ and which satisfies certain natural assumptions. Some applications are also obtained. Finally, we are concerned with extreme points and support points associated with certain extension operators that preserve Loewner chains.

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Key words. Carathéodory family, extreme point, Loewner chain, parametric representation, subordination, support point.

REFERENCES

[1] Arosio, L., *Resonances in Loewner equations*, Adv. Math., **227** (2011), 1413–1435.

[2] Arosio, L., Bracci, F., Hamada, H. and Kohr, G., *An abstract approach to Loewner chains*, J. Anal. Math., **119** (2013), 89–114.

[3] Arosio, L., Bracci, F. and Wold, F.E., *Solving the Loewner PDE in complete hyperbolic starlike domains of* $\mathbb{C}^n$, Adv. Math., **242** (2013), 209–216.

[4] Bracci, F., Contreras, M.D. and Madrigal, S.-Díaz., *Evolution families and the Loewner equation II: complex hyperbolic manifolds*, Math. Ann., **344** (2009), 947–962.

[5] Chirilă, T., *Subclasses of biholomorphic mappings associated with g-Loewner chains on the unit ball in* $\mathbb{C}^n$, Complex Var. Elliptic Equ., DOI:10.1080/17476933.2013.856422, to appear.

[6] Duren, P., Graham, I., Hamada, H. and Kohr, G., *Solutions for the generalized Loewner differential equation in several complex variables*, Math. Ann., **347** (2010), 411–435.

[7] Elin, M., *Extension operators via semigroups*, J. Math. Anal. Appl., **377** (2011), 239–250.

[8] Graham, I., Hamada, H. and Kohr, G., *Parametric representation of univalent mappings in several complex variables*, Canad. J. Math., **54** (2002), 2, 324–351.

[9] Graham, I., Hamada, H. and Kohr, G., *Radius problems for holomorphic mappings on the unit ball in* $\mathbb{C}^n$, Math. Nachr., **279** (2006), 1474–1490.

[10] Graham, I., Hamada, H., Kohr, G. and Kohr, M., *Extreme points, support points and the Loewner variation in several complex variables*, Sci. China Math., **55** (2012), 1353–1366.

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[11] Graham, I., Hamada, H., Kohr, G. and Kohr, M., *Extremal properties associated with univalent subordination chains in* $\mathbb{C}^n$, Math. Ann., DOI 10.1007/s00208-013-0998-y, to appear.
[12] Graham, I. and Kohr, G., *Univalent mappings associated with the Roper-Suffridge extension operator*, J. Anal. Math., **81** (2000), 331–342.
[13] Graham, I. and Kohr, G., *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
[14] Graham, I., Kohr, G. and Kohr, M., *Loewner chains and the Roper-Suffridge extension operator*, J. Math. Anal. Appl., **247** (2000), 448–465.
[15] Graham, I., Kohr, G. and Kohr, M., *Loewner chains and parametric representation in several complex variables*, J. Math. Anal. Appl., **281** (2003), 425-438.
[16] Graham, I., Kohr, G. and Pfaltzgraff, J.A., *Parametric representation and linear functionals associated with extension operators for biholomorphic mappings*, Rev. Roum. Math. Pures Appl., **52** (2007), 47–68.
[17] Hallenbeck, D.J. and MacGregor, T.H., *Linear Problems and Convexity Techniques in Geometric Function Theory*, Pitman, Boston, 1984.
[18] Hamada, H., *Polynomially bounded solutions to the Loewner differential equation in several complex variables*, J. Math. Anal. Appl., **381** (2011), 179–186.
[19] Hamada, H. and Honda, T., *Sharp growth theorems and coefficient bounds for starlike mappings in several complex variables*, Chinese Ann. Math. Ser.B., **29** (2008), 353–368.
[20] Hamada, H., Honda, H. and Kohr, G., *Growth theorems and coefficient bounds for univalent holomorphic mappings which have parametric representation*, J. Math. Anal. Appl., **317** (2006), 302–319.
[21] Kirwan, W.E., *Extremal properties of slit conformal mappings*, in *Aspects of Contemporary Complex Analysis* (Brannan, D. and Clunie, J., eds.), Academic Press, London-New York, 1980, 439–449.
[22] Kirwan, W.E. and Schober, G., *New inequalities from old ones*, Math. Z., **180** (1982), 19–40.
[23] Muir, J.R., *A class of Loewner chain preserving extension operators*, J. Math. Anal. Appl., **337** (2008), 862–879.
[24] Pell, R., *Support point functions and the Loewner variation*, Pacific J. Math., **86** (1980), 561–564.
[25] Pfaltzgraff, J.A., *Subordination chains and univalence of holomorphic mappings in* $\mathbb{C}^n$, Math. Ann., **210** (1974), 55–68.
[26] Pfaltzgraff, J.A. and Suffridge, T.J., *An extension theorem and linear invariant families generated by starlike maps*, Ann. Univ. Mariae Curie-Sklodowska, Sect. A. **53** (1999), 193–207.
[27] Pommerenke, Ch., *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
[28] Poreda, T., *On the univalent holomorphic maps of the unit polydisc in* $\mathbb{C}^n$ *which have the parametric representation, I-the geometrical properties*, Ann. Univ. Mariae Curie-Sklodowska Sect. A., **41** (1987), 105–113.
[29] Poreda, T., *On the univalent holomorphic maps of the unit polydisc in* $\mathbb{C}^n$ *which have the parametric representation, II-the necessary conditions and the sufficient conditions*, Ann. Univ. Mariae Curie-Sklodowska Sect. A., **41** (1987), 115–121.
[30] Range, M., *Holomorphic Functions and Integral Representations in Several Complex Variables*, Springer-Verlag, New York, 1986.
[31] Roper, K. and Suffridge, T.J., *Convex mappings on the unit ball of* $\mathbb{C}^n$, J. Anal. Math., **65** (1995), 333–347.
[32] Roth, O., *Control Theory in* $\mathcal{H}(\mathbb{D})$, Diss. Univ. Würzburg, 1998.
[33] Schleissinger, S., *On support points of the class* $S^0(B^n)$, Proc. Amer. Math. Soc., to appear.

[34] Suffridge, T.J., *Starlikeness, convexity and other geometric properties of holomorphic maps in higher dimensions*, Lecture Notes Math., **599**, Springer-Verlag, 1977, 146–159.
[35] Voda, M., *Solution of a Loewner chain equation in several complex variables*, J. Math. Anal. Appl., **375** (2011), 58–74.

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