

ON EXTENDED CONVERGENCE DOMAINS
FOR THE NEWTON-KANTOROVICH METHOD

IOANNIS K. ARGYROS and SANTHOSH GEORGE

Abstract. We present results on extended convergence domains and their applications for the Newton-Kantorovich method (NKM), using the same information as in previous papers. Numerical examples are provided to emphasize that our results can be applied to solve nonlinear equations using (NKM), in contrast with earlier results which are not applicable in these cases.

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Cameron University

Department of Mathematical Sciences

Lawton, OK 73505, USA

E-mail: iargyros@cameron.edu

National Institute of Technology Karnataka

Department of Mathematical and

Computational Sciences

India-757 025

E-mail: sgeorge@nitk.ac.in