

COMMON FIXED POINT THEOREMS
FOR WEAKLY COMPATIBLE MAPPINGS IN MENGER SPACES
VIA COMMON LIMIT RANGE PROPERTY

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Abstract. Inspired by the well-known concept of common limit range property due to Sintunavarat and Kumam, we prove common fixed point theorems for two pairs of weakly compatible mappings in Menger spaces. Some illustrative examples are given in order to demonstrate the validity of our main result. We extend our results to four finite families of self-mappings.

MSC 2010. 47H10, 54H25.

Key words. t-norm, Menger space, weakly compatible mappings, common limit range property, common fixed point.

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Received February 19, 2012
Accepted March 5, 2014

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