MATHEMATICA, Tome 55 (78), N° 2, 2013, pp. 125–141

ULAM STABILITY OF A CUBIC FUNCTIONAL EQUATION IN VARIOUS SPACES

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Abstract. We prove the Hyers-Ulam-Rassias stability of the cubic functional equation

$$f(x+my)+f(x-my)=2(2\cos(\tfrac{m\pi}{2})+m^2-1)f(x) \\ -\tfrac{1}{2}(\cos(\tfrac{m\pi}{2})+m^2-1)f(2x)+m^2(f(x+y)+f(x-y))$$

in various spaces.

MSC 2010. 39B52, 39B72, 39B82.

Key words. Cubic functional equation, Hyers-Ulam-Rassias stability, non-Archimedean normed space, quasi-Banach space, random normed space.

REFERENCES

[1] Aoki, T., *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan., **2** (1950), 64–66.

[2] Arunkumar, M., Rassias, M.J. and Zhang, Y., *Ulam-Hyers stability of a 2-variable AC-mixed type functional equation: direct and fixed point methods*, Journal of Modern Mathematics Frontier, **1** (2012), 10–26.

[3] Baktash, E., Cho, Y.J., Jalili, M., Saadati, R., and Vaezpour, S.M., *On the stability of cubic mappings and quadratic mappings in random normed spaces*, J. Inequal. Appl., (2008), Article ID 902187, DOI 10.1155/2008/902187, 11 pages.

[4] Benyamini,Y. and Lindenstrauss, J., *Geometric Nonlinear Functional Analysis*, Vol. 1, American Mathematical Society Colloquium Publications, **48**, American Mathematical Society, Providence, RI, 2000.

[5] Bodaghi, A., *Cubic derivations on Banach algebras*, Acta Math. Vietnam., **38** (2013), 517–528.

[6] Bodaghi, A. and Alias, I.A., *Approximate ternary quadratic derivations on ternary Banach algebras and C^*-ternary rings*, Adv. Difference Equ., (2012), Article No. 11.

[7] Bodaghi, A., Alias, I.A. and Ghahramani, M.H., *Approximately cubic functional equations and cubic multipliers*, J. Inequal. Appl., (2011), Article No. 53.

[8] Bodaghi, A., Moosavi, S.M. and Rahimi, H., *The generalized cubic functional equation and the stability of cubic Jordan $*$-derivations*, Ann. Univ. Ferrara Sez. VII Sci. Mat, **59** (2013), 235–250.

[9] Czerwik, S., *On the stability of the quadratic mapping in normed spaces*, Abh. Math. Sem. Univ. Hambg., **62** (1992), 59–64.

[10] Eshaghi Gordji, M., Bodaghi, A. and Park, C., *A fixed point approach to the stability of double Jordan centralizers and Jordan multipliers on Banach algebras*, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., **73** (2011), 65–73.

The author expresses his sincere thanks to the reviewer for the careful and detailed reading of the manuscript and for the helpful suggestions that improved the manuscript substantially.

[11] Eshaghi Gordji, M., Kaboli Gharetapeh, S., Savadkouhi, M.B., Aghaei, M. and Karimi, T., *On cubic derivations*, Int. J. Math. Anal., **4** (2010), 51, 2501–2514.
[12] Eskandani, G.Z. and Rassias, J.M., *Approximation of a general cubic functional equation in Felbin's type fuzzy normed linear spaces*, Results Math., 2014; DOI 10.1007/s00025-014-0367-5.
[13] Eskandani, G.Z., Rassias, J.M. and Găvruţă, P., *Generalized Hyers-Ulam stability for a general cubic functional equation in quasi-β-normed spaces*, Asian-Eur. J. Math., **4** (2011); DOI 10.1142/S1793557111000332.
[14] P. Găvruţă, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184** (1994), 431–436.
[15] Hensel, K., *Über eine neue Begründung der Theorie der algebraischen Zahlen*, Jahresber. Deutsch. Math.-Verein., **6** (1897), 83–88.
[16] Hyers, D.H., *On the stability of the linear functional equation,* Proc. Natl. Acad. Sci. USA, **27** (1941), 222–224.
[17] Hyers, D.H., Isac, G. and Rassias, Th.M., *Stability of Functional Equations in Several Variables,* Birkhäuser, Basel, 1998.
[18] Jun, K.W. and Kim, H.M., *The generalized Hyers-Ulam-Russias stability of a cubic functional equation*, J. Math. Anal. Appl., **274** (2002), 867–878.
[19] Jun, K.W. and Kim, H.M., *On the Hyers-Ulam-Rassias stability of a general cubic functional equation*, Math. Inequal. Appl., **6** (2003), 289–302.
[20] Miheţ, D. and Radu, V., *On the stability of the additive Cauchy functional equation in random normed spaces*, J. Math. Anal. Appl., **343** (2008), 567–572.
[21] Moslehian, M.S. and Rassias, Th.M., *Stability of functional equations in non-Archimedean spaces*, Appl. Anal. Discrete Math., **1** (2007), 325–334.
[22] Najati, A., *Hyers-Ulam-Rassias stability of a cubic functional equation*, Bull. Korean Math. Soc., **44** (2007), 825–840.
[23] Najati, A., *The generalized Hyers-Ulam-Rassias stability stability of a cubic functional equation,* Turkish J. Math., **31** (2007), 395–408.
[24] Park, C. and Bodaghi, A., *On the stability of $*$-derivations on Banach $*$-algebras*, Adv. Difference Equ., (2012), Article No. 138.
[25] Rassias, J.M., *Solution of the Ulam stability problem for cubic mappings*, Glas. Mat. Ser. III, **36 (56)** (2001), 63–72.
[26] Rassias, Th.M., *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72** (1978), 297–300.
[27] Ravi, K., Rassias, J.M. and Narasimman, P., *Stability of a cubic functional equation in fuzzy normed space*, Journal of Applied Analysis and Computation, **1** (2011), 411–425.
[28] Rolewicz, S., *Metric Linear Spaces*, second edition, PWN–Polish Scientific Publishers, Warsaw, D. Reidel Publishing Co., Dordrecht, 1984.
[29] Schweizer, B. and Sklar, A., *Probabilistic Metric Spaces*, Elsevier, North Holand, New York, 1983.
[30] Šerstnev, A.N., *On the motion of a random normed space*, Dokl. Akad. Nauk, **149** (1963), 280-283.
[31] Tabor, J., *stability of the Cauchy functional equation in quasi-Banach spaces*, Ann. Polon. Math., **83** (2004), 243–255.
[32] Ulam, S.M., *Problems in Modern Mathematics,* Chapter VI, Science Ed., Wiley, New York, 1940.
[33] Xu, T.Z. and Rassias, J.M., *On the Hyers-Ulam stability of a general mixed additive and cubic functional equation in n-Banach spaces*, Abstr. Appl. Anal., 2012, Article ID 926390, 23 pages.

Received June 18, 2013
Accepted March 27, 2014

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