

LAYER POTENTIAL ANALYSIS OF A NEUMANN PROBLEM
FOR THE BRINKMAN SYSTEM

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Abstract. In this paper we obtain the existence and uniqueness result (up to a constant pressure) in some Sobolev spaces for a Neumann problem associated to the Brinkman system on Lipschitz domains in compact boundaryless Riemannian manifolds. In order to obtain the desired result, we use an indirect boundary integral formulation based on the potential theory for the Brinkman system.

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Key words. Brinkman system, existence and uniqueness, Lipschitz domain, layer potential theory, boundary integral equations, Sobolev spaces.

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