

THE METHOD OF LOEWNER CHAINS IN THE STUDY  
OF THE UNIVALENCE OF  $C^2$  MAPPINGS

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**Abstract.** We continue the work of W.C. Royster [26], P.T. Mocanu [20, 21], M. Cristea [4-7], G. Kohr [19], H. Hamada and G. Kohr [14] of extending univalence criteria for holomorphic mappings to  $C^1$  mappings and we continue our work from [7] of improving the method of Loewner chains which is used in complex univalence theory. We show that the method remains valid even for  $C^2$  mappings which are not necessarily holomorphic and we give further applications of our results.

**MSC 2010.** Primary 32H02; Secondary 30C45.

**Key words.** Loewner chain, Loewner differential equation, univalent mapping, quasiconformal mapping.

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