

## IDEMPOTENTS IN GROUP ALGEBRAS

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**Abstract.** In this survey we collect and present the classical and some recent methods to compute the primitive (central) idempotents in semisimple group algebras.

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**Key words.** Idempotents, group algebras, group rings, Wedderburn decomposition, group codes.

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