

COPURE (m, n) -INJECTIVE MODULES AND
 (α, m, n) -COTORSION MODULES

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Abstract. In this paper we introduce the notions of copure (m, n) -injective modules and (α, m, n) -cotorsion modules as generalizations of (m, n) -injective modules and (m, n) -cotorsion modules, respectively. We also obtain some properties of these modules.

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