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POTENTIAL ANALYSIS FOR PSEUDODIFFERENTIAL MATRIX OPERATORS IN LIPSCHITZ DOMAINS ON RIEMANNIAN MANIFOLDS. APPLICATIONS TO BRINKMAN OPERATORS

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Abstract. In this paper we present the main properties of layer potentials associated to some pseudodifferential matrix type operators on Lipschitz domains in compact Riemannian manifolds of arbitrary dimension. We focus on a class of Brinkman operators and show compactness and invertibility results of associated layer potential operators, and well-posedness results for related transmission problems with the boundary data in some Sobolev spaces.

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Key words. Brinkman operator, Lipschitz domain, Riemannian manifold, layer potential operator, compactness, invertibility, transmission problem, well-posedness.

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