

NEARLY PARTIAL TERNARY CUBIC DERIVATIONS
ON BANACH TERNARY ALGEBRAS

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Abstract. Let A_1, A_2, \dots, A_n be normed ternary algebras over the complex field \mathbb{C} and let B be a Banach ternary algebra over \mathbb{C} . A mapping δ_k from $A_1 \times \dots \times A_n$ into B is called a k -th partial ternary cubic derivation if there exists a cubic mapping $g_k : A_k \rightarrow B$ such that

$$\begin{aligned} \delta_k(x_1, \dots, [a_k b_k c_k], \dots, x_n) &= [g_k(a_k)g_k(b_k)\delta_k(x_1, \dots, c_k, \dots, x_n)] \\ &+ [g_k(a_k)\delta_k(x_1, \dots, b_k, \dots, x_n)g_k(c_k)] + [\delta_k(x_1, \dots, a_k, \dots, x_n)g_k(b_k)g_k(c_k)], \end{aligned}$$

and

$$\begin{aligned} &\delta_k(x_1, \dots, 2a_k + b_k, \dots, x_n) + \delta_k(x_1, \dots, 2a_k - b_k, \dots, x_n) \\ &= 2\delta_k(x_1, \dots, a_k + b_k, \dots, x_n) \\ &+ 2\delta_k(x_1, \dots, a_k - b_k, \dots, x_n) + 12\delta_k(x_1, \dots, a_k, \dots, x_n), \end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ ($i \neq k$). We prove the generalized Hyers-Ulam-Rassias stability of the partial ternary cubic derivations on Banach ternary algebras.

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Key words. Hyers-Ulam-Rassias stability, generalized Hyers-Ulam-Rassias, Banach ternary algebra, partial ternary cubic derivation.

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