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ULAM STABILITY GENERALIZATIONS OF 4^{th} ORDER TERNARY DERIVATIONS ASSOCIATED TO A JMRASSIAS QUARTIC FUNCTIONAL EQUATION ON FRÉCHET ALGEBRAS

ALI EBADIAN, NOROUZ GHOBADIPOUR, TAHEREH RASTAD, and MEYSAM BAVAND SAVADKOUHI

Abstract. Let $\mathcal{A}$ be a Banach ternary algebra over $\mathbb{R}$ or $\mathbb{C}$ and $\mathcal{X}$ be a ternary Banach $\mathcal{A}$-module. A quartic mapping $D : (\mathcal{A}, [\,]_{\mathcal{A}}) \to (\mathcal{X}, [\,]_{\mathcal{X}})$ is called a 4^{th} order ternary derivation if $D([x,y,z]) = [D(x), y^4, z^4] + [x^4, D(y), z^4] + [x^4, y^4, D(z)]$ for all $x, y, z \in \mathcal{A}$. We prove Ulam stability generalizations of 4^{th} order ternary derivations associated to the following JMRassias quartic functional equation on Fréchet algebras $f(2x+y)+f(2x-y) = 4f(x+y)+4f(x-y)+24f(x)-6f(y)$.

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Key words. Ulam stability, quartic functional equation, ternary Banach algebras, Fréchet algebras, 4^{th} order ternary derivation.

REFERENCES

[1] Abramov, V., Kerner, R. and Le Roy, B., *Hypersymmetry: A Z_3 graded generalization of supersymmetry,* J. Math. Phys., **38** (1997), 1650–1669.

[2] Aczel, J. and Dhombres, J., *Functional Equations in Several Variables,* Cambridge Univ. Press, 1989.

[3] Bavand Savadkouhi, M., Eshaghi Gordji, M., Rassias, J.M., and Ghobadipour, N., *Approximate ternary Jordan derivations on Banach ternary algebras,* J. Math. Phys., **50** (2009).

[4] Bazunova, N., Borowiec, A. and Kerner, R., *Universal differential calculus on ternary algebras,* Lett. Math. Phys., **67** (2004), 195–206.

[5] Cayley, A., *On the* 34 *concomitants of the ternary cubic,* Amer. J. Math., **4** (1881), 1–15.

[6] Cholewa, P.W., *Remarks on the stability of functional equations,* Aequationes Math., **27** (1984), 76–86.

[7] Chu, H., Koo, S. and Park, J., *Partial stabilities and partial derivations of n-variable functions,* Nonlinear Anal., **72** (2010), 1531–1541.

[8] Chung, J.K. and Sahoo, P., K., *On the general solution of a quartic functional equation,* Bull. Korean Math. Soc., **40** (2003), 565–576.

[9] Czerwik, S., *On the stability of the quadratic mapping in normed spaces,* Abh. Math. Semin. Univ. Hambg., **62** (1992), 59–64.

[10] Ebadian, A., Najati, A. and Eshaghi Gordji, M., *On approximate additive–quartic and quadratic–cubic functional equations in two variables on abelian groups,* Results Math., **58** (2010), 39–53.

[11] Ebadian, A., Ghobadipour, N. and Eshaghi Gordji, M., *A fixed point method for perturbation of bimultipliers and Jordan bimultipliers in C^*-ternary algebras,* J. Math. Phys., **51**, 103508 (2010).

[12] Ebadian, A., Ghobadipour, N., Banand Savadkouhi, M., and Eshaghi Gordji, M., *Stability of a mixed type cubic and quartic functional equation in non-Archimedean ℓ-fuzzy normed spaces,* Thai J . Math., **9** (2011), 225–241.

[13] Ebadian, A., Ghobadipour, N., Rassias, Th.M., and Eshaghi Gordji, M., *Functional inequalities associated with Cauchy additive functional equation in non-Archimedean spaces,* Discrete Dyn. Nat. Soc., **2011** (2011), Article ID 929824.

[14] Ebadian, A., Ghobadipour, N., Rassias, Th.M., and Nikoufar, I., *Stability of generalized derivations on Hilbert C^*- modules associated to a pexiderized Cauchy-Jensen type functional equation,* Acta Math. Sci . Ser. B Engl. Ed., **32B(3)** (2012), 1226–1238.

[15] Eshaghi Gordji, M., Ebadian, A. and Zolfaghari, S., *Stability of a functional equation deriving from cubic and quartic functions,* Abstr. Appl. Anal., **2008** (2008), Article ID 801904.

[16] Eshaghi Gordji, M., *Stability of an additive-quadratic functional equation of two variables in Fspaces,* J. Nonlinear Sci. Appl., **2** (2009), 251–259.

[17] Eshaghi Gordji, M. and Ghobadipour, N., *Nearly generalized Jordan derivations,* Math. Slovaca, **61** (2011), 1–8.

[18] Eshaghi Gordji, M. and Ghobadipour, N., *Approximately quartic homomorphisms on Banach algebras,* World Applied Sciences Journal, in press.

[19] Eshaghi Gordji, M. and Ghobadipour, N., *Stability of $(\alpha, \beta, \gamma)-$derivations on Lie C^*-algebras,* Int. J. Geom. Methods Mod. Phys., **7** (2010), 1093–1102.

[20] Eshaghi Gordji, M., Rassias, J.M. and Ghobadipour, N., *Generalized Hyers–Ulam stability of the generalized (n, k)–derivations,* Abstr. Appl. Anal., **2009** (2009), Article ID 437931.

[21] Gajda, Z., *On stability of additive mappings,* Int. J. Math. Math. Sci., **14** (1991), 431–434.

[22] Găvruţa, P., *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings,* J. Math. Anal. Appl., **184** (1994), 431–436.

[23] Găvruţa, P., *An answer to a question of Th.M. Rassias and J. Tabor on mixed stability of mappings,* Bul. Ştiinţ. Univ. Politeh. Timiş. Ser. Mat. Fiz., **4** (56) (1997), 1–6.

[24] Găvruţa, P., *On the Hyers–Ulam–Rassias stability of mappings,* in: *Recent Progress in Inequalities*, vol. **430**, Kluwer, 1998, pp. 465–469.

[25] Ghobadipour, N., *Lie *-double derivations on Lie C^*-algebras,* International Journal of Nonlinear Analysis and Applications, **1** (2010), 1–12.

[26] Ghobadipour, N., Ebadian, A., Rassias, Th.M., and Eshaghi, M., *A perturbation of double derivations on Banach algebras,* Commun. Math. Anal. **11** (2011), 51–60.

[27] Hyers, D.H., Isac, G. and Rassias, Th.M., *Stability of Functional Equations in Several Variables,* Birkhäuser, Basel, 1998.

[28] Hyers, D.H., *On the stability of the linear functional equation,* Proc. Natl. Acad. Sci. USA, **27** (1941), 222–224.

[29] Isac, G. and Rassias, Th.M., *On the Hyers–Ulam stability of ψ-additive mappings,* J. Approx. Theory, **72** (1993), 131–137.

[30] Kapranov, M., Gelfand, I.M. and Zelevinskii, A., *Discrimininants, Resultants and Multidimensional Determinants,* Birkhäuser, Berlin, 1994.

[31] Kerner, R., *Ternary algebraic structures and their applications in physics,* Univ. P.M. Curie preprint, Paris (2000), http://arxiv.org/list/math-ph/0011.

[32] Kerner, R., *The cubic chessboard,* Classical Quantum Gravity, **14** (1997), A203-A225.

[33] Moslehian, M.S., *Almost derivations on C*-ternary rings,* Bull. Belg. Math. Soc. Simon Stevin, **14** (2007), 135–142.

[34] Moslehian, M.S., *Ternary derivations, stability and physical aspects,* Acta Appl. Math, **100** (2008), 187–199.

[35] Park, C. and Eshaghi Gordji, M., *Comment on Approximate ternary Jordan derivations on Banach ternary algebras [Bavand Savadkouhi et al. J. Math. Phys. 50, 042303 (2009)]*, J. Math. Phys. **51**, 044102 (2010).
[36] Park, W.G. and Bae, J.H., *On a bi-quadratic functional equation and its stability,* Nonlinear Anal., **62** (2005) 643–654.
[37] Rassias, J.M., *Solution of the Ulam stability problem for quartic mappings,* Glas. Mat. Ser. III, **34** (1999), 243–252.
[38] Rassias, J.M., *On a new approximation of approximately linear mappings by linear mappings,* Discuss. Math., **7** (1985), 193–196.
[39] Rassias, J.M., *On the stability of the Euler-Lagrange functional equation,* Chin. J. Math., **20** (1992), 185–190.
[40] Rassias, J.M., *On approximation of approximately linear mappings by linear mappings*, Bull. Sci. Math., **108** (1984), 445–446.
[41] Rassias, J.M., *On approximation of approximately linear mappings by linear mappings,* J. Funct. Anal., **46** (1982), 126–130.
[42] Rassias, J.M., *Solution of a problem of Ulam,* J. Approx. Theory, **57** (1989), 268–273.
[43] Rassias, Th.M., *On the stability of the linear mapping in Banach spaces,* Proc. Amer. Math. Soc., **72** (1978), 297–300.
[44] Rassias, Th. M., *On the stability of the quadratic functional equation and its applications,* Stud. Univ. Babeş-Bolyai, Math., **43** (1998), 89-124.
[45] Rassias, Th.M., *The problem of S.M. Ulam for approximately multiplicative mappings,* J. Math. Anal. Appl., **246** (2000), 352-378.
[46] Rassias, Th.M. and Tabor, J., *What is left of Hyers-Ulam stability?*, J. Nat. Geom., **1** (1992), 65–69.
[47] Saadati, R., Cho, Y.J. and Vahidi, J., *The stability of the quartic functional equation in various spaces,* Comput. Math. Appl., **60** (2010), 1994–2002.
[48] Sewell, G.L.,*Quantum Mechanics and its Emergent Macrophysics,* Princeton Univ. Press, Princeton, NJ, 2002.
[49] Skof, F., *Proprietà locali e approssimazione di operatori,* Rend. Semin. Mat. Fis. Milano, **53** (1983), 113–129.
[50] Ulam, S.M., *Problems in modern mathematics,* Chapter VI, Science ed., Wiley, New York, 1940.
[51] Zettl, H., *A characterization of ternary rings of operators,* Adv. Math., **48** (1983), 117–143.

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Urmia University
Department of Mathematics
Urmia, Iran
E-mail: a.ebadian@urmia.ac.ir
E-mail: ghobadipour.n@gmail.com
E-mail: bavand.m@gmail.com

Islamic Azad University
Zarrindasht Branch
Zarrindasht, Iran
E-mail: rastad.tahereh@gmail.com