

AN EXTENSION OPERATOR AND LOEWNER CHAINS  
ON THE EUCLIDEAN UNIT BALL IN  $\mathbb{C}^n$

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**Abstract.** We are concerned with an extension operator  $\Phi_{n,\alpha}$ ,  $\alpha \geq 0$ , that provides a way of extending a locally biholomorphic mapping  $f \in H(B^n)$  to a locally biholomorphic mapping  $F \in H(B^{n+1})$ . In the case  $\alpha = 1/(n+1)$ , this operator reduces to the Pfaltzgraß-Suffridge extension operator. By using the method of Loewner chains, we prove that if  $f \in S^0(B^n)$ , then  $\Phi_{n,\alpha}(f) \in S^0(B^{n+1})$ , whenever  $\alpha \in [0, 1/(n+1)]$ . In particular, if  $f \in S^*(B^n)$ , then  $\Phi_{n,\alpha}(f) \in S^*(B^{n+1})$ , and if  $f$  is spirallike of type  $\beta \in (-\pi/2, \pi/2)$  on  $B^n$ , then  $\Phi_{n,\alpha}(f)$  is also spirallike of type  $\beta$  on  $B^{n+1}$ . We also prove that if  $f$  is almost starlike of order  $\beta \in [0, 1)$  on  $B^n$ , then  $\Phi_{n,\alpha}(f)$  is almost starlike of order  $\beta$  on  $B^{n+1}$ . Finally we prove that if  $f \in K(B^n)$  and  $1/(n+1) \leq \alpha \leq 1/n$ , then the image of  $F = \Phi_{n,\alpha}(f)$  contains the convex hull of the image of some egg domain contained in  $B^{n+1}$ . An extension of this result to the case of  $\varepsilon$ -starlike mappings will be also considered.

**MSC 2010.** 32H02, 30C45.

**Key words.** Biholomorphic mapping, convex mapping, Loewner chain, parametric representation, Pfaltzgraß-Suffridge extension operator, Roper-Suffridge extension operator, starlike mapping,  $\varepsilon$ -starlike mapping.

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