MATHEMATICA, Tome 54 (77), N° 2, 2012, pp. 103–116

AN INTERMEDIATE NEWTON ITERATIVE SCHEME AND GENERALIZED ZABREJKO-NGUEN AND KANTOROVICH EXISTENCE THEOREMS FOR NONLINEAR EQUATIONS

IOANNIS K. ARGYROS and LIVINUS U. UKO

Abstract. We revisit a one-step intermediate Newton iterative scheme that was used by Uko and Velásquez in [17] for the constructive solution of nonlinear equations of the type $f(u) + g(u) = 0$. By utilizing weaker hypotheses of the Zabrejko-Nguen kind and a modified majorizing sequence we perform a semilocal convergence analysis which yields finer error bounds and more precise information on the location of the solution that the ones obtained in [17]. We also give two generalizations of the well-known Kantorovich theorem on the solvability of nonlinear equations and the convergence of Newton's method. Illustrative examples are provided in the paper.

MSC 2010. 65H99, 49M15.

Key words. Nonlinear equations, Newton's method, intermediate Newton method, Zabrejko-Nguen conditions, iterative solution, majorant method, majorizing sequence, Lipschitz condition, center-Lipschitz condition.

REFERENCES

[1] Appel, J., De Pascale, E., Evkuta, N.A. and Zabrejko, P.P., *On the two-step Newton method for the solution of nonlinear operator equations*, Math. Nachr., **172** (1995), 5–14.

[2] Argyros, I.K., *Quadratic equations and applications to Chandrasekhar's and related equations*, Bull. Aust. Math. Soc., **32** (1985), 275–292.

[3] Argyros, I.K., *On a class of nonlinear integral equations arising in neutron transport*, Aequationes Math., **36** (1988), 99–111.

[4] Argyros, I.K., *On the midpoint method for solving nonlinear operator equations and applications to the solution of integral equations*, Rev. Anal. Numér. Théor. Approx., **23** (1994), 139–152.

[5] Argyros, I.K., *Polynomial operator equations in abstract spaces and applications*, CRC Press LLC, Boca Raton, Florida, USA, 1998.

[6] Argyros, I.K., *On a Multistep Newton Method in Banach Spaces and the Pták Error Estimates*, Adv. Nonlinear Var. Inequal., **6** (2003), 121–135.

[7] Argyros,I.K., *On the Newton-Kantorovich hypothesis for solving equations*, J. Comput. Appl. Math., **11**, (2004), 103–110.

[8] Argyros, I.K., *A unifying local-semilocal convergence analysis and applications for two-point Newton-like methods in Banach spaces*, J. Math. Anal. Appl., **298** (2004), 374–397.

[9] Chandrasekhar, S., *Radiactive transfer*, Dover Publ. New York, 1960.

[10] Dennis, J.E., *On the Kantorovich hypotheses for Newton's method*, SIAM J. Numer. Anal., **6** (1969), 493–507.

[11] Gragg, W.B. and Tapia, R.A., *Optimal error bounds for the Newton-Kantorovich theorem*, SIAM J. Numer. Anal., **11** (1974), 10–13.

[12] Kantorovich, L.V., *On Newton's method for functional equations* (Russian), Dokl. Akad. Nauk., **59** (1948), 1237–1240.
[13] Nagatou, K., Yamamoto, N. and Nakao, M.T., *An approach to the numerical verification of solutions for nonlinear elliptic problems with local uniqueness*, Numer. Funct. Anal. Optim., **20** (1999), 543–565.
[14] Ortega, L.M. and Rheinboldt, W.C., *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, New York, 1970.
[15] Ostrowski, A.M., *Solution of Equations in Euclidean and Banach Spaces*, Academic Press, New York, 1973.
[16] Potra, F.A., *The Kantorovich Theorem and interior point methods*, Math. Program., **102** (2005), 47–70.
[17] Uko, L.U. and Velásquez Ossa, R.E., *Convergence Analysis of a one-step intermediate Newton iterative scheme*, Rev. Colombiana Mat., **35** (2001), 21–27.
[18] Yamamoto, T., *Error bounds for Newton's process derived from the Kantorovich conditions*, Japan J. Appl. Math., **2** (1985), 285–292.
[19] Zabrejko, P.P. and Nguen, D.F., *The majorant method in the theory of Newton-Kantorovich Approximations and the Ptak error estimates*, Numer. Funct. Anal. Optim., **9** (1987), 671–684.

Received November 24, 2011
Accepted October 22, 2012

Cameron University
Department of Mathematical Sciences
Lawton, OK 73505, USA
E-mail: iargyros@cameron.edu

Georgia Gwinnett College
Department of Science and Technology
Decator, Georgia 30033–4097, USA
E-mail: luko@ggc.edu