

SUR L'APPLICATION D'ABEL-JACOBI,
L'ESPACE DES MODULES DES SURFACES DE RIEMANN
ET LE PROBLÈME DE SCHOTTKY

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Abstract. In this paper we discuss some interesting problems. Abel's theorem classifies divisors by their images in the jacobian. The Jacobi inversion problem asks whether we can find a divisor that is the preimage for an arbitrary point in the jacobian. The Schottky problem is the problem of characterizing jacobian varieties among all abelian varieties. We study the map X (Riemann surface) to its jacobian variety $\text{Jac}(X)$ from a moduli point of view. The problem consists in finding an analytical characterization of the complex tori that arise as jacobians inside the Siegel upper half space.

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