

STABILITY OF AQ-FUNCTIONAL EQUATIONS IN  
NON-ARCHIMEDEAN  $\mathcal{L}$ -FUZZY NORMED SPACES

MADJID ESHAGHI GORDJI, MEYSAM BAVAND SAVADKOUHI,  
and ASGHR TAHERI SARTESHNIZI

**Abstract.** In this paper we prove the generalized Hyers-Ulam stability of the mixed type additive and quadratic functional equation

$$f(3x + y) + f(3x - y) = f(x + y) + f(x - y) + 2f(3x) - 2f(x)$$

in non-Archimedean  $\mathcal{L}$ -fuzzy normed spaces.

**MSC 2010.** 39B82, 39B52.

**Key words.** Stability, quadratic functional equation, non-Archimedean  $\mathcal{L}$ -fuzzy normed space.

REFERENCES

- [1] ACZEL, J. and DHOMBRES, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, 1989.
- [2] AOKI, T., *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan., **2** (1950), 64–66.
- [3] CHOLEWA, P.W., *Remarks on the stability of functional equations*, Aequationes Math., **27** (1984), 76–86.
- [4] CZERWIK, S., *On the stability of the quadratic mapping in normed spaces*, Abh. Math. Sem. Univ. Hamburg, **62** (1992), 59–64.
- [5] DESCHRIJVER, G. and KERRE, E., *On the relationship between some extensions of fuzzy set theory*, Fuzzy Sets and Systems, **133** (2003), 227–235.
- [6] EBADIAN, A., NAJATI A. and GORDJI, M.E., *On approximate additive-quartic and quadratic-cubic functional equations in two variables on abelian groups*, Results Math., **58** (2010), 39–53.
- [7] ESHAGHI GORDJI, M., *Stability of a functional equation deriving from quartic and additive functions*, Bull. Korean Math. Soc., **47** (2010), 491–502.
- [8] ESHAGHI GORDJI, M., *Stability of an additive-quadratic functional equation of two variables in  $F$ -spaces*, J. Nonlinear Sci. Appl., **2** (2009), 251–259.
- [9] ESHAGHI GORDJI, M., *Nearly ring homomorphisms and nearly ring derivations on non-Archimedean Banach algebras*, Abs. Appl. Anal., **2010**, Article ID 393247, 12 pages.
- [10] ESHAGHI GORDJI, M. and BAVAND SAVADKOUHI, M., *Stability of cubic and quartic functional equations in non-Archimedean spaces*, Acta Appl. Math., **110** (2010), 1321–1329.
- [11] ESHAGHI GORDJI, M. and BAVAND SAVADKOUHI, M., *Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces*, Appl. Math. Letters, **23** (2010), 1198–1202.
- [12] ESHAGHI GORDJI, M. and BAVAND SAVADKOUHI, M., *On approximate cubic homomorphisms*, Adv. Difference Equ., **2009**, Article ID 618463, 11 pages.
- [13] ESHAGHI GORDJI, M., EBADIAN, A. and ZOLFAGHARI, S., *Stability of a functional equation deriving from cubic and quartic functions*, Abstr. Appl. Anal., **2008**, Article ID 801904, 17 pages.

- [14] ESHAGHI GORDJI, M., GHAEMI, M.B., KABOLI GHARETAPEH, S., SHAMS, S. and EBA-  
DIAN, A., *On the stability of  $J^*$ -derivations*, J. Geom. Phys., **60** (2010), 454–459.
- [15] ESHAGHI GORDJI, M., GHAEMI, M.B., and MAJANI, H., *Generalized Hyers-Ulam-  
Rassias theorem in Menger probabilistic normed spaces*, Discrete Dyn. Nat. Soc., **2010**,  
Article ID 162371, 11 pages.
- [16] ESHAGHI GORDJI, M., GHAEMI, M.B., MAJANI, H. and PARK, C., *Generalized Ulam-  
Hyers stability of Jensen functional equation in Šerstnev PN spaces*, J. Ineq. Appl.,  
**2010**, Article ID 868193, 14 pages.
- [17] ESHAGHI GORDJI M., KHODABAKHSH, R., JUNG, S.M. and KHODAEI, H., *AQCQ-  
functional equation in non-Archimedean normed spaces*, Abs. Appl. Anal., **2010**, Article  
ID 741942, 22 pages.
- [18] ESHAGHI GORDJI, M. and KHODAEI, H., *On the generalized Hyers-Ulam-Rassias sta-  
bility of quadratic functional equations*, Abs. Appl. Anal., **2009**, Article ID 923476, 11  
pages.
- [19] ESHAGHI GORDJI, M. and KHODAEI, H., *Stability of Functional Equations*, LAP LAM-  
BERT Academic Publishing, 2010.
- [20] ESHAGHI GORDJI, M. and KHODAEI, H., *Solution and stability of generalized mixed  
type cubic, quadratic and additive functional equation in quasi-Banach spaces*, Nonlinear  
Anal.-TMA, **71** (2009), 5629–5643.
- [21] ESHAGHI GORDJI, M., KHODAEI H. and KHODABAKHSH, R., *General quartic-cubic-  
quadratic functional equation in non-Archimedean normed spaces*, U.P.B. Sci. Bull. (Se-  
ries A), **72** (2010), 69–84.
- [22] ESHAGHI GORDJI, M. and NAJATI, A., *Approximately  $J^*$ -homomorphisms: a fixed point  
approach*, J. Geom. Phys., **60** (2010), 809–814.
- [23] GAJDA, Z., *On stability of additive mappings*, Internat. J. Math. Math. Sci., **14** (1991),  
431–434.
- [24] GÁVRUTA, P., *A generalization of the Hyers-Ulam-Rassias stability of approximately  
additive mappings*, J. Math. Anal. Appl., **184** (1994), 431–436.
- [25] GOGUEN, J., *L-fuzzy sets*, Math. Anal. Appl., **18** (1967), 145–174.
- [26] GORDJI, M.E., ZOLFAGHARI, S., RASSIAS, J.M. and SAVADKOUHI, M.B., *Solution and  
stability of a mixed type cubic and quartic functional equation in quasi-Banach spaces*,  
Abs. Appl. Anal., **2009**, Article ID 417473, 14 pages.
- [27] GORDJI, M.E., ABBASZADEH, S. and PARK, C., *On the stability of generalized mixed  
type quadratic and quartic functional equation in quasi-Banach spaces*, J. Inequal. Appl.,  
**2009** (2009), Article ID 153084, 26 pages.
- [28] GORDJI, M.E., KABOLI-GHARETAPEH, S., PARK, C. and ZOLFAGHRI, S., *Stability of an  
additive-cubic-quartic functional equation*, Adv. Difference Equ., **2009** (2009), Article  
ID 395693, 20 pages.
- [29] GORDJI, M.E., KABOLI GHARETAPEH, S., RASSIAS, J.M. and ZOLFAGHARI, S., *So-  
lution and stability of a mixed type additive, quadratic and cubic functional equation*,  
Adv. Difference Equ., **2009**, Article ID 826130, 17 pages.
- [30] GORDJI, M.E. and SAVADKOUHI, M.B., *Stability of a mixed type cubic and quartic  
functional equations in random normed spaces*, J. Inequal. Appl., **2009** (2009), Article  
ID 527462, 9 pages.
- [31] GORDJI, M.E., SAVADKOUHI, M.B. and PARK, C. *Quadratic-quartic functional equa-  
tions in RN-spaces*, J. Inequal. Appl., **2009** (2009), Article ID 868423, 14 pages.
- [32] GORDJI, M.E. and SAVADKOUHI, M.B., *Approximation of generalized homomorphisms  
in quasi-Banach algebras*, An. Științ. Univ. “Ovidius” Constanța, **17** (2009), 203–214.
- [33] GORDJI, M.E., SAVADKOUHI, M.B. and BIDKHAM, M., *Stability of a mixed type additive  
and quadratic functional equation in non-Archimedean spaces*, J. Comput. Anal. Appl.,  
**12** (2010), 454–462.

- [34] GRABIEC, A., *The generalized Hyers-Ulam stability of a class of functional equations*, Publ. Math. Debrecen., **48** (1996), 217–235.
- [35] HENSEL, K., *Über eine neue Begründung der Theorie der algebraischen Zahlen*, Jahresber. Deutsch. Math.-Verein., **6** (1897), 83–88.
- [36] HYERS, D.H., ISAC, G. and RASSIAS, TH.M., *Stability of functional equations in several variables*, Birkhäuser, Basel, 1998.
- [37] HYERS, D.H., *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA, **27** (1941), 222–224.
- [38] ISAC, G. and RASSIAS, TH. M., *On the Hyers-Ulam stability of  $\psi$ -additive mappings*, J. Approx. Theory, **72** (1993), 131–137.
- [39] JUNG, S.M., *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Palm Harbor, 2001.
- [40] KANNAPPAN, PL., *Quadratic functional equation and inner product spaces*, Results Math., **27** (1995), 368–372.
- [41] NAJATI, A. and RASSIAS, TH.M., *Pexider type operators and their norms in  $X_\lambda$  spaces*, Czechoslovak Math. J., **59 (134)** (2009), 1087–1094.
- [42] NAJATI, A. and ZAMANI ESKANDANI, G., *Stability of a mixed additive and cubic functional equation in quasi-Banach spaces*, J. Math. Anal. Appl., **342** (2008), 1318–1331.
- [43] RASSIAS, J.M., *On Approximation of approximately linear mappings by linear mappings*, J. Funct. Anal., **46** (1982), 126–130.
- [44] RASSIAS, J.M., *On Approximation of approximately linear mappings by linear mappings*, Bull. Sci. Math., **108** (1984), 445–446.
- [45] RASSIAS, J.M., *Solution of a problem of Ulam*, J. Approx. Theory., **57** (1989), 268–273.
- [46] RASSIAS, J.M., *On the stability of the Euler-Lagrange functional equation*, Chinese J. Math., **20** (1992), 185–190.
- [47] RASSIAS, J.M., *Solution of a stability problem of Ulam*, Discuss. Math., **12** (1992), 95–103.
- [48] RASSIAS, J. M., *Complete solution of the multi-dimensional problem of Ulam*, Discuss. Math., **14** (1994), 101–107.
- [49] RASSIAS, TH.M., *Functional Equations Inequalities and Applications*, Kluwer Academic Publishers, Dordrecht, Boston and London, 2003.
- [50] RASSIAS, TH.M., *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72** (1978), 297–300.
- [51] SKOF, F., *Proprietá locali e approssimazione di operatori*, Rend. Sem. Mat. Fis. Milano, **53** (1983), 113–129.
- [52] SHAKERI, S., SAADATI, R. and PARK, C., *Stability of the quadratic functional equation in non-Archimedean  $\mathcal{L}$ -fuzzy normed spaces*, Int. J. Nonlin. Anal. Appl, **1** (2010), 1–12.
- [53] ULAM, S.M., *Problems in Modern Mathematics*, Science Ed., Wiley, New York, 1940.

Received August 8, 2010

Accepted January 16, 2011

*Semnan University*  
*Department of Mathematics*  
*P.O. Box 35195-363, Semnan, Iran*  
*CENAA, Semnan University, Iran*  
*E-mail: madjid.eshaghi@gmail.com*  
*E-mail: bavand.m@gmail.com*

*Payame Noor University*  
*Department of Mathematics*  
*Broojen Branch, Broojen, Iran*  
*E-mail: taheri2028@gmail.com*