MATHEMATICA, Tome 54 (77), N° 1, 2012, pp. 16–25

IDEMPOTENT-NILPOTENT UNITS OF COMMUTATIVE GROUP RINGS

PETER DANCHEV

Abstract. Suppose that R is a commutative unital ring and G is a multiplicative abelian group. We find a criterion when the decomposition of normalized invertible elements $V(RG) = Id(RG) \times (1 + I(N(R)G; G))$ holds. In particular, when $supp(G) \cap inv(R) \neq \emptyset$, we establish such a necessary and sufficient condition only in terms of R and G. This strengthens a result due to Karpilovsky (Arch. Math., 1983) as well as results of the author (Bull. Greek Math. Soc., 2009), (Comm. Algebra, 2010) and (Comment. Math. Univ. Carolin., 2012).

MSC 2010. 16S34, 16U60, 20K10, 20K20, 20K21.

Key words. Group rings, unit groups, decompositions, nilpotents, idempotents.

REFERENCES

[1] Danchev, P., *On idempotent units in commutative group rings*, An. Univ. Bucureşti Mat., **58** (2009), 17–22.
[2] Danchev, P., *Idempotent-nilpotent units in commutative group rings*, Bull. Greek Math. Soc., **56** (2009), 21–28.
[3] Danchev, P., *Trivial units in abelian group algebras*, Extracta Math., **24** (2009), 47–53.
[4] Danchev, P., *Idempotent units of commutative group rings*, Comm. Algebra, **38** (2010), 4649–4654.
[5] Danchev, P., *G-nilpotent units of commutative group rings*, Comment. Math. Univ. Carolin., **53** (2012).
[6] Karpilovsky, G., *On units in commutative group rings*, Arch. Math. (Basel), **38** (1982), 420–422.
[7] Karpilovsky, G., *On finite generation of unit groups of commutative group rings*, Arch. Math. (Basel), **40** (1983), 503–508.
[8] Karpilovsky, G., *Unit Groups of Group Rings*, Longman Sci. and Techn., Harlow, 1989.
[9] Karpilovsky, G., *Units of commutative group algebras*, Expo. Math., **8** (1990), 247–287.
[10] May, W., *Group algebras over finitely generated rings*, J. Algebra, **39** (1976), 483–511.
[11] Mollov, T. and Nachev, N., *Group of normalized units of commutative modular group rings*, Ann. Sci. Math. Québec, **33** (2009), 83–92.

Received December 03, 2010
Accepted March 22, 2011

Plovdiv State University
Department of Mathematics
Plovdiv 4000, Bulgaria
E-mail: pvdanchev@yahoo.com

The author would like to thank the referee for the careful reading of the manuscript.