

CONVOLUTION TYPE OPERATORS WITH OSCILLATING
SYMBOLS ON WEIGHTED LEBESGUE SPACES
ON A UNION OF INTERVALS

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Abstract. We establish Fredholm criteria for convolution type operators W with oscillating symbols, continuous on \mathbb{R} and admitting mixed (slowly oscillating and semi-almost periodic) discontinuities at $\pm\infty$, on weighted Lebesgue spaces on a union of intervals with weights in a subclass of Muckenhoupt weights.

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