# THE SECOND HANKEL DETERMINANT $H_2(n)$
# FOR ODD STARLIKE AND CONVEX FUNCTIONS

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**Abstract.** For odd starlike and convex functions $f$ defined on the open unit disk $\mathbb{U}$, the upper bounds of the functional $|a_n a_{n+2} - a_{n+1}^2|$, defined by using the second Hankel determinant $H_2(n)$ due to J. W. Noonan and D. K. Thomas (see [4]), are studied. Furthermore, applying the second Hankel determinant $H_2(n)$, a new operator $\mathcal{H}$ is introduced and the properties of new functions $\mathcal{H}f$ are discussed.

**MSC 2010.** 34C40.

**Key words.** Hankel determinant, odd analytic function, odd starlike function, odd convex function.

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