

ON STRONGLY STARLIKE AND CONVEX FUNCTIONS
OF ORDER α AND TYPE β

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Abstract. In this note we investigate the inclusion relationship between the class of strongly starlike functions of order α and type β , $\alpha \in (0, 1]$ and $\beta \in [0, 1)$, which satisfy

$$\left| \arg \left\{ \frac{zf'(z)}{f(z)} - \beta \right\} \right| < \frac{\pi}{2} \alpha$$

and the class of strongly convex functions of order α and type β which satisfy

$$\left| \arg \left\{ 1 + \frac{zf''(z)}{f'(z)} - \beta \right\} \right| < \frac{\pi}{2} \alpha$$

in the unit disk, where f is an analytic function defined on the unit disk and satisfies $f(0) = f'(0) - 1 = 1$. Some applications of our main result are also presented which contains various classical results for the typical subclasses of starlike and convex functions.

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Key words. Univalent function, strongly starlike function, strongly convex function.

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