

MODULES WITH DEDEKIND FINITE ENDOMORPHISM RINGS

SIMION BREAZ, GRIGORE CĂLUGĂREANU and PHILL SCHULTZ

Abstract. This article is a survey of modules whose endomorphism rings are Dedekind finite, Hopfian or co-Hopfian. We summarise the properties of such modules and present unified proofs of known results and generalisations to new structure theorems.

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Key words. Dedekind finite, Hopfian, co-Hopfian, endomorphism ring.

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“Babeş-Bolyai” University
Faculty of Mathematics and Computer Science
Str. Mihail Kogălniceanu nr. 1
400084 Cluj-Napoca, Romania
E-mail: bodo@math.ubbcluj.ro
E-mail: calug2001@yahoo.com

The University of Western Australia
School of Mathematics and Statistics
6009 Nedlands, Australia
E-mail: schultz@maths.uwa.edu.au