

ON A k -COMPLEX MOMENT PROBLEM

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Abstract. In this paper we give a necessary and sufficient condition on a sequence $\{\Gamma_{\alpha,\beta} = (s_{ij}(\alpha,\beta))_{1 \leq i,j \leq k}, \alpha, \beta \in \mathbb{N}^n\}_{\alpha,\beta}$ of (k, k) matrices with complex entries, $k \in \mathbb{N}^*$, to be a complex moment sequence with respect to a (k, k) positive defined matrix of Borel measures on the unit polydisc. The proof in this note is different from the proof of a similar result in [Theorem 1.4.8, 19] in case that $\Gamma_{\alpha,\beta}$ are bounded operators acting on an arbitrary Hilbert space, with $\Gamma_{\alpha,\beta} = \Gamma_{\beta,\alpha}^*$. The proof in this note also omits the condition $\Gamma_{\alpha,\beta} = \Gamma_{\beta,\alpha}^*$ on the sequence of matrices $\{\Gamma_{\alpha,\beta}\}_{\alpha,\beta \in \mathbb{N}^n}$ from the hypothesis of [Theorem 1.4.8, 19].

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