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ON THE DETERMINATION OF 3D AUTONOMOUS FORCE FIELDS PRODUCING TRAJECTORIES THAT ARE SOLUTIONS OF A SYSTEM OF ODES

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Abstract. This study is a generalization of a recent work by Bozis and Borghero (2008) establishing connections between autonomous planar force fields and the entire two-parametric set of solutions of a given linear second order ODE (solvable or not). In this paper we find 3D force fields which give rise to a *three-parametric family of spatial orbits*. It is shown that the three-parametric set of all solutions of any system of *linear* ordinary differential equations of the type $y''(x) = f_0(x)+yf_1(x)+zf_2(x)+y'f_3(x)$, $z'(x) = g_0(x)+yg_1(x)+zg_2(x)+y'g_3(x)$ (which may be solvable by quadratures or not) represents a set of regular orbits traced by a material point of unit mass, in the presence of *at least* one autonomous force field $\bar{F}(X, Y, Z)$, for adequate initial conditions. The corresponding force field is determined by quadratures on the grounds of the eight functions $f_i(x)$, $g_k(x)$ $(i, k{=}0,1,2,3)$ which specify the above system of ODEs. Subcases are also studied and pertinent examples are offered.

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Key words. Inverse Problems, 3D autonomous force fields, three-parametric families of orbits, ODEs and PDEs.

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