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ON NON-NORMALIZED SUBORDINATION CHAINS IN $\mathbb{C}^n$

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Abstract. In this paper we consider non-normalized univalent subordination chains $f(z,t) = \exp(\int_0^t A(\tau)\mathrm{d}\tau)z + \cdots$ and we present the connection with the notion of generalized A-asymptotic spirallikeness on the Euclidean unit ball B^n in $\mathbb{C}^n$, where $A : [0,\infty) \to L(\mathbb{C}^n, \mathbb{C}^n)$ is a measurable operator that satisfies certain natural conditions.

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Key words. Biholomorphic mapping, generalized asymptotic spirallike mapping, Loewner differential equation, subordination, subordination chain.

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