

HOPF-GALOIS EXTENSIONS
AND ISOMORPHISMS OF SMALL CATEGORIES

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Abstract. We associate two linear categories with two objects to a module over the subalgebra of coinvariants of a Hopf-Galois extension, and prove that they are isomorphic. The structure Theorem for cleft extensions, and the Militaru-Ştefan lifting Theorem can be obtained using these isomorphisms.

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Key words. Hopf-Galois extension, Morita equivalence, Picard group, cleft extension, Sweedler cohomology.

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