

LAGRANGE'S EQUILATERAL TRIANGULAR CONFIGURATION:
COMPUTATION AND ANALYSIS
OF THE RIEMANNIAN CURVATURES

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Abstract. In this paper we study the Riemannian curvatures for the triangular equilateral configuration of Lagrange in the case of the planar three-body problem. The curvature tensor is computed using symbolic computation systems and a result regarding the sign of sectional curvatures is proven.

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Key words. Three body problem, Lagrange configuration, Riemannian curvature.

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