

PRE-SCHWARZIAN NORM ESTIMATES OF FUNCTIONS  
FOR A SUBCLASS OF STRONGLY STARLIKE FUNCTIONS

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**Abstract.** For normalized analytic functions  $f$  in the unit disk  $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ , we consider

$$\mathcal{S}^*(\alpha, \beta) = \left\{ f : \frac{zf'(z)}{f(z)} \prec \left( \frac{1 + (1 - 2\beta)z}{1 - z} \right)^\alpha, z \in \mathbb{D} \right\},$$

where  $0 < \alpha \leq 1$  and  $0 \leq \beta < 1$ . There exists a close connection between Bloch functions and univalent functions. In this paper, we present an optimal, but not sharp, estimate of the Bloch semi-norm of the function  $\log f'$  for  $f \in \mathcal{S}^*(\alpha, \beta)$ .

**MSC 2010.** Primary 30C45; Secondary 30C55, 33C05.

**Key words.** Pre-Schwarzian derivative, univalent, starlike and strongly starlike functions, subordination.

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