

ON MAPPING PROPERTIES OF LAYER POTENTIAL
OPERATORS FOR BRINKMAN EQUATIONS ON
LIPSCHITZ DOMAINS IN RIEMANNIAN MANIFOLDS

MIRELA KOHR, CORNEL PINTEA, and WOLFGANG L. WENDLAND

Abstract. In this paper we present the main properties of layer potential operators for general Brinkman equations on Lipschitz domains in compact Riemannian manifolds. These properties are used to obtain an existence and solvability result in Sobolev-Slobodetski spaces for a transmission problem given in terms of two general Brinkman operators, when the solution is defined in two complementary Lipschitz or C^1 domains on a Riemannian manifold and satisfies prescribed transmission conditions at the interface between these domains.

MSC 2010. Primary 35J25; Secondary 42B20, 46E35, 76D, 76M.

Key words. Brinkman operator, Lipschitz and C^1 domains, Riemannian manifold, transmission problem, layer potential operators.

REFERENCES

- [1] COSTABEL, M., *Boundary integral operators on Lipschitz domains: Elementary results*, SIAM J. Math. Anal., **19** (1988), 613–626.
- [2] DINDOŠ, M. and MITREA, M., *The stationary Navier-Stokes system in nonsmooth manifolds: the Poisson problem in Lipschitz and C^1 domains*, Arch. Ration. Mech. Anal. **174** (2004), 1–47.
- [3] FABES, E., KENIG, C. and VERCHOTA, G., *The Dirichlet problem for the Stokes system on Lipschitz domains*, Duke Math. J., **57** (1988), 769–793.
- [4] GAGLIARDO, E., *Proprietà di alcune classi di funzioni in piu variabili*, Ricerche Mat., **7** (1958), 102–137.
- [5] GESZTESY, F. and MITREA, M., *Robin-to-Robin maps and Krein-type resolvent formulas for Schrödinger operators on bounded Lipschitz domains*. In: Modern Analysis and Applications: Mark Krein Centenary Conference, **191** (2009), 81–113.
- [6] HSIAO, G.C. and WENDLAND, W.L., *Boundary Integral Equations* Springer, 2008.
- [7] KOHR, M., PINTEA, C. and WENDLAND, W.L., *Stokes-Brinkman transmission problems on Lipschitz and C^1 domains in Riemannian manifolds*, Commun. Pure Appl. Anal., **9** (2010), 493–537.
- [8] KOHR, M., PINTEA, C. and WENDLAND, W.L., *Brinkman-type operators on Riemannian manifolds: Transmission problems in Lipschitz and C^1 domains*, Potential Anal., to appear. DOI 10.1007/s11118-009-9151-7.
- [9] KOHR, M. and POP, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press, Southampton (UK), 2004.

- [10] KOHR, M., RAJA SEKHAR, G.P. and WENDLAND, W.L., *Boundary integral equations for a three-dimensional Stokes-Brinkman cell model*, Math. Models Methods Appl. Sci., **18** (2008), 2055–2085.
- [11] KOHR, M. and WENDLAND, W.L., *Boundary integral equations for a three-dimensional Brinkman flow problem*, Math. Nachr., **282** (2009), 1305–1333.
- [12] MITREA, D., MITREA, M. and QIANG, S., *Variable coefficient transmission problems and singular integral operators on non-smooth manifolds*, J. Integral Equations Appl., **18** (2006), 361–397.
- [13] MITREA, D., MITREA, M. and TAYLOR, M., *Layer Potentials, the Hodge Laplacian and Global Boundary Problems in Non-Smooth Riemannian Manifolds*, Mem. Amer. Math. Soc., **150**, No. 713, 2001.
- [14] MITREA, M. and TAYLOR, M., *Boundary layer methods for Lipschitz domains in Riemannian manifolds*, J. Funct. Anal., **163** (1999), 181–251.
- [15] MITREA, M. and TAYLOR, M., *Potential theory on Lipschitz domains in Riemannian manifolds: Sobolev-Besov space results and the Poisson problem*, J. Funct. Anal., **176** (2000), 1–79.
- [16] MITREA, M. and TAYLOR, M., *Navier-Stokes equations on Lipschitz domains in Riemannian manifolds*, Math. Anal., **321** (2001), 955–987.
- [17] MITREA, M. and TAYLOR, M., *Potential theory on Lipschitz domains in Riemannian manifolds: the case of Dini metric tensors*, Trans. Amer. Math. Soc., **355** (2003), 1961–1985.
- [18] MITREA, M. and WRIGHT, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, to appear.
- [19] TAYLOR, M., *Pseudodifferential Operators*, Princeton Univ. Press, Princeton, 1981.
- [20] TAYLOR, M., *Partial Differential Equations*, Springer, New York, 1996–1997, vols. 1–3.
- [21] WLOKA, J.T., ROWLEY, B. and LAWURK, B., *Boundary Value Problems for Elliptic Systems*, Cambridge Univ. Press, Cambridge, 1995.

Received July 20, 2009

Accepted September 5, 2009

“Babeş-Bolyai” University
Faculty of Mathematics and Computer Science
Str. Mihail Kogălniceanu Nr. 1
400084 Cluj-Napoca, Romania
E-mail: mkohr@math.ubbcluj.ro
E-mail: cpintea@math.ubbcluj.ro

Universität Stuttgart
Institut für Angewandte Analysis
und Numerische Simulation
Pfaffenwaldring 57
70569 Stuttgart, Germany
E-mail: wendland@mathematik.uni-stuttgart.de