AN IMPROVED CONVERGENCE ANALYSIS OF NEWTON'S METHOD FOR SYSTEMS OF EQUATIONS WITH CONSTANT RANK DERIVATIVES

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Abstract. We use Newton's method to solve systems of equations with constant rank derivatives. Motivated by optimization considerations, and using more precise estimates, we provide a convergence analysis for Newton's method with the following advantages over the work in [11]: larger convergence domain; finer error estimates on the distances involved, and an at least as precise information on the location of the solution. These improvements are obtained under the same hypotheses and computational cost as in [11]. Kantorovich-type as well as Smale-type point estimate applications are also provided.

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Key words. Newton's method, constant rank derivatives, semilocal convergence, Lipschitz condition with L average.

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