

INTEGRAL CHARACTERIZATIONS OF
WEIGHTED BLOCH SPACES AND $Q_{K,\omega}(p, q)$ SPACES

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Abstract. In this paper we introduce a new space of functions, the so called $Q_{K,\omega}(p, q)$ space of holomorphic functions on the unit disk in terms of nondecreasing functions. The relation between the integral norm of the $Q_{K,\omega}(p, q)$ space and the integral norm of the weighted Bloch space $\mathcal{B}_\omega^\alpha$ is also given. Further, we obtain similar integral criteria for the little weighted Bloch functions of analytic functions and meromorphic functions.

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