

THE FARTHEST POINT PROBLEM IN NON-ARCHIMEDEAN
NORMED SPACES

MOHAMMAD SAL MOSLEHIAN, ASSADOLLAH NIKNAM and SEDDIGHEH
SHADKAM

Abstract. We study the farthest point mapping in non-Archimedean normed spaces. We prove that a uniquely remotal subset M in a non-Archimedean normed space X is singleton if for some Chebyshev center c and some $|\alpha| < 1$ the equality $q_M(\alpha c + (1 - \alpha)q_M(c)) = q_M(c)$ holds. We show that M is singleton if and only if $\|x - q_M(x)\| = \|y - q_M(y)\|$ implies that $q_M(x) = q_M(y)$. We also prove that if X, Y are non-Archimedean normed spaces and $Z = X \times Y$ is equipped with the norm $\|(x, y)\| = \max\{|x|, |y|\}$, then all uniquely remotal sets in $(Z, \|\cdot\|)$ are singletons.

MSC 2000. Primary 46S10; secondary 41A65, 46B20.

Key words. Farthest point, Chebyshev center, uniquely remotal set, normed space, non-Archimedean normed space, non-Archimedean field.

REFERENCES

- [1] BARONTI, M., *A note on remotal sets in normed spaces*, Publ. Inst. Math., Nouv. Sér., **53** (67) (1993), 95–98.
- [2] BOSZNAY, A.P., *A remark on the farthest point problem I*, J. Approx. Theory, **27** (1979), 309–312.
- [3] HENSEL, K., *Über eine neue Begründung der Theorie der algebraischen Zahlen*, Jahresber. Deutsch. Math. Verein, **6** (1897), 83–88.
- [4] KHRENNIKOV, A., *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models*, Kluwer Academic Publishers, Dordrecht, 1997.
- [5] MIRMOSTAFAEI, A.K. and NIKNAM, A., *A remark on uniquely remotal sets*, Indian J. Pure Appl. Math., **29** (1998), 849–854.
- [6] MOSLEHIAN, M.S. and RASSIAS, TH.M., *Stability of functional equations in non-Archimedean spaces*, Appl. Anal. Disc. Math., **1** (2007), 325–334.
- [7] MOSLEHIAN, M.S. and SADEGHI, G., *A Mazur–Ulam theorem in non-Archimedean normed spaces*, Nonlinear Anal., **69** (2008), 3405–3408.
- [8] NIKNAM, A., *On uniquely remotal sets*, Indian J. Pure Appl. Math., **15** (1984), 1079–1083.
- [9] VAN ROOIJ, A.C.M., *Non-Archimedean functional analysis*, Monographs and Textbooks in Pure and Applied Math., 51. Marcel Dekker, New York, 1978.

Received January 17, 2008

Accepted August 26, 2008

Ferdowsi University of Mashhad
Department of Mathematics
Centre of Excellence in Analysis
on Algebraic Structures (CEAAS)
P. O. Box 1159
Mashhad 91775, Iran

E-mail: moslehian@ferdowsi.um.ac.ir

E-mail: niknam@math.um.ac.ir,

dassamankin@yahoo.co.uk

E-mail: shadkam.s@wali.um.ac.ir