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COEFFICIENT ESTIMATES AND THE CONVEX HULL PROBLEM FOR MEROMORPHIC FUNCTIONS

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Abstract. We consider the class $S(p)$ of meromorphic univalent functions in the unit disk $\mathbb{D}$ having a simple pole at $p \in (0,1)$. Let $\Sigma^s(p,w_0)$ consist of functions $f \in S(p)$ for which $\overline{\mathbb{C}} \setminus f(\mathbb{D})$ is a starlike set with respect to a point $w_0 \neq 0, \infty$. In this paper, we find a sharp estimate for the real part of the constant coefficient in the Laurent expansion of functions in $S(p)$. Also we prove a result on the closed convex hull of $\Sigma^s(p,w_0)$. Lastly, we obtain certain coefficient estimates in the Laurent expansion for functions in $\Sigma^s(p,w_0)$.

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