

MULTIPLICITY RESULTS FOR NONLINEAR EIGENVALUE  
PROBLEMS ON UNBOUNDED DOMAINS

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**Abstract.** In this paper we prove a multiplicity result for a class of eigenvalue problems with nonlinear boundary conditions on an unbounded domain. Many results have been obtained by Cârstea and Rădulescu [3], Chabrowski [5], [6], Kandilakis and Lyberopoulos [10], Lisei, Varga and Horváth [13] and Pflüger [16].

**MSC 2000.** 35J60, 35P30, 58E05.

**Key words.**  $p(x)$ -Laplacian, critical points theory.

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Received October 10, 2008

Accepted November 25, 2008

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