

A NOTE ON ANNIHILATORS AND INJECTIVITY

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**Abstract.** It is proved that every two-sided ideal of a ring  $A$  is generated by a central idempotent if and only if every two-sided ideal of  $A$  is the left and right annihilator of an element of  $A$  and the intersection of the Jacobson radical, the left singular ideal and the right singular ideal of  $A$  is zero. The following generalization of injective modules, distinct from  $p$ -injective modules, is studied: a left  $A$ -module  $M$  is said to satisfy (\*) if, for any left submodule  $N$  of  $M$  isomorphic to a complement left submodule  $C$  of  $M$ , every left  $A$ -monomorphism of  $N$  into  $C$  extends to a left  $A$ -homomorphism of  $M$  into  $C$ .

**MSC 2000.** 16D50, 16E50, 16P20.

**Key words.** Annihilator, von Neumann regular, module satisfying (\*), continuous regular,  $p$ -injective module, singular submodule.

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