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TWO-DIMENSIONAL POTENTIALS GENERATING A GIVEN ONE-PARAMETER FAMILY OF ORBITS ON A SURFACE

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Abstract. We say that a potential *generates a curve* on a surface if a unit mass traces the curve under the action of the potential. We consider the following problem: a one-parameter family of regular curves $f(u,v)=c$ on a surface $\vec{r}(u,v)=\{x(u,v),\ y(u,v),\ z(u,v)\}$ is given. We seek two-dimensional potentials of the form $V(u,v)=u^m R(\frac{v}{u})$, R being an arbitrary C^2-function, which generate this family of regular curves as trajectories on the above surface. We show that if the given family of orbits satisfies exactly two differential conditions, then such a potential exists and it is determined uniquely. Special cases are also studied and pertinent examples are given for each case. At a second step, if we consider that the "*slope function*" $\gamma(u,v)=f_v/f_u$ is homogeneous of zero degree and the components of the metric tensor are homogeneous functions of zero degree too, then a potential of the above form always exists and it is found as a solution of an ordinary second-order O.D.E. Several examples are offered and implications of this study are discussed.

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